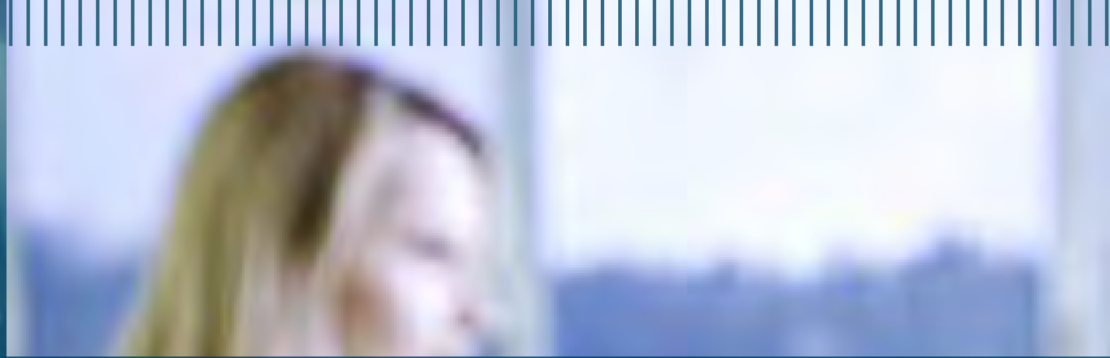

THE PROFESSIONAL RISK MANAGER (PRM™)
CERTIFICATION PROGRAMME

PRMIA
The Professional Risk Managers' International Association

PRM Self Study Guide – Exam II

(TWO OF FIVE)

MATHEMATICAL FOUNDATIONS OF RISK MEASUREMENT

PRM™ SELF-STUDY GUIDE – EXAM II

Mathematical Foundations of Risk Measurement

OVERVIEW

Exam II of the PRM™ certification tests a candidate's knowledge and understanding of the mathematical foundations of risk measurement.

In Exam II, we take you through the mathematical foundations of risk assessment. While there are many nuances to the practice of risk management that go beyond the quantitative, it is essential today for every risk manager to be able to assess risks. The chapters in this section are accessible to all PRM™ members, including those without any quantitative skills. The Excel spreadsheets that are available with Volume III of the PRM™ Handbook are an invaluable aid to understanding the mathematical and statistical concepts that form the basis of risk assessment.

You can use this Self-Study Guide to focus your study on the key Learning Outcome Statements from each chapter. These Learning Outcome Statements form the basis for the questions asked during the examination that you will take as Exam II of the PRM™ certification program. We recommend that you first read the chapter, then review the Learning Outcome Statements, then re-read the chapter with particular emphasis on these points.

We recommend strongly that you do not simply read the Learning Outcome Statements and then try to find the information about each in the books as a short-cut way of preparing for the exam. Real-life risk management requires your ability to assemble information from many simultaneous inputs and you can expect that some exam questions will draw from multiple Learning Outcome Statements.

After studying the book for this section, becoming comfortable with your knowledge and understanding of each Learning Outcome Statement, and working through the Study Questions and the Sample Exam Questions, you will have read the materials necessary for passing Exam II of the PRM™ Certification program. You may then wish to purchase access to online Sample Exams (Diagnostics) via the PRMIA website to assess your readiness.

Taking the PRM™ qualification, as well as working as a risk officer, requires a certain amount of mathematical expertise. This is not excessive. Anyone who was passed mathematics studies at advanced high school level, or who has completed the first year of a university degree in a mathematical-based qualification (physics, economics, engineering, etc) should have no problem with the requirements. For others, we recommend that they take tuition in the mathematics required and that they focus on this as the first part of their studies for the PRM™.

Please note that testing conditions, your state of mind and various factors can make your performance on the actual exams somewhat less strong than on the Sample Exams. If your Sample Exam scores are near to the passing mark, you may wish to study the subject materials even further.

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Please remember that the exams of the PRM™ certification are very challenging. After all it's "a higher standard in risk certification" and you would expect nothing less. There is no guarantee that using the Self-Study Guide, in combination with the reading materials and Sample Exams will give you a passing score. But, they should all provide you with assistance in doing your best. We wish you much success in your effort to become certified as a Professional Risk Manager!

WORD DEFINITIONS

In this guide, we use the Command Words that the CFA Institute uses, and a few additional words, to indicate levels of ability expected from successful candidates on each Learning Outcome Statement.

Calculate	To ascertain or determine by mathematical processes.
Characterize	To describe the essential character or quality of.
Compare	To examine the character or qualities of, for the primary purpose of discovering resemblances.
Construct	To create by organizing ideas or concepts logically and coherently.
Contrast	To compare in respect to differences.
Deconstruct	To disassemble the key elements of ideas or concepts.
Define	To set forth the meaning of; specifically, to formulate a definition of.
Demonstrate	To prove or make clear by reasoning or evidence; to illustrate and explain, especially with examples.
Derive	To obtain by reasoning.
Describe	To transmit a mental image, an impression, or an understanding of the nature and characteristics of.
Differentiate	To mark or show a difference in; to develop different characteristics in.
Discuss	To discourse about through reasoning or argument; to present in detail.
Draw	To express graphically in words; to delineate.
Explain	To give the meaning or significance of; to provide an understanding of; to give the reason for or cause of.
Identify	To establish the identity of; to show or prove the sameness of.
List	To enumerate.
Show	To set forth in a statement, account, or description; to make evident or clear.
State	To express in words.

STUDY TIME

Preparation time will vary greatly according to your knowledge and understanding of the subject matter prior to your self-study, your ability to commit dedicated and uninterrupted time to your study and other factors. In general, candidates who prepare for the exams of the PRM™ certification program allocate about three months to preparation for each exam.

You may spend three hours each week in study, or as much as ten or more, each week to ready yourself. Follow the suggestions above regarding the use of the Learning Outcome Statements and Sample Exams. Once you are comfortable with your readiness, it's time to register for the exam.

TESTING STRATEGIES

All questions are multiple-choice and there are no penalties for incorrect answers. Bear in mind that it is vitally important to finish the exam in the time allotted. Do not linger over questions longer than is sensible.

For example, if the exam has 30 questions in 90 minutes, do not spend longer than three minutes per question. If at the end of three minutes you have not answered the question, decide on the best answer you can (ignoring the obviously wrong), mark your answer and move on. If you do have any spare time at the end of the exam you can always go back and review the answer. However, make absolutely sure that you have an answer for every question at the end of the exam!

Another strategy would be to go through all the questions answering the ones you find easier ones first. Then after a first pass, divide the remaining questions by the time remaining and proceed as above.

USAGE OF THE CALCULATOR

At the exam centre you will have access to an on-line calculator. This is very similar to the Microsoft Calculator (available as standard with Microsoft Windows) and offers both scientific and standard functionality which is toggled through a "View" menu option.

Candidates may **not** take their own calculators into the exam venue. Candidates must familiarize themselves with the usage of this calculator before the exam including all calculations described in the PRM™ Handbooks. Finding out during the exam that the exam venue calculator is not the same as that with which you are familiar is not a recommended strategy!

STUDY QUESTIONS

A few questions, with answers, have been provided to help the candidate understand some of the concepts of the PRM™ Handbook. These study questions are not comprehensive of all concepts in the exam, nor are they necessarily questions of a similar type to those in the exam. They are provided in good faith as a study aid.

SAMPLE EXAM QUESTIONS

In Volume 5 of this Study Guide there are sample questions from all for exams. These sample questions should give you a flavor for the format and content of the actual exams. They are only part of the length of the actual exams and therefore do not cover all subjects contained in the detailed content description provided in this document. Questions on any of the subjects listed previously may appear on the actual exam.

MATHEMATICAL FOUNDATIONS OF RISK MEASUREMENT

Financial risks cannot be properly managed unless they are quantified. And the assessment of risk requires mathematics. During the last decade value-at-risk (VaR) has become the ubiquitous tool for risk capital estimation. To understand a VaR model, risk managers require knowledge of probability distributions, simulation methods and a host of other mathematical and statistical techniques. Even if not directly responsible for designing and coding a risk capital model, middle office risk managers must understand the Market VaR, Credit VaR and Operational VaR models sufficiently well to be competent to assess them. The middle office risk manager's responsibility has expanded to include the independent validation of trader's models, as well as risk capital assessment. And the role of risk management in the front office itself has expanded, with the need to hedge increasingly complex options portfolios.

So today, the hallmark of a good risk manager is not just having the statistical skills required for risk assessment – a comprehensive knowledge of pricing and hedging financial instruments is equally important. The PRM™ qualification includes an entire exam on mathematical and statistical methods.

However, we do recognize that many students will not have degrees in mathematics, physics or other quantitative disciplines. So, Volume II of the PRM™ Handbook is aimed at students having no quantitative background at all. It introduces and explains all the mathematics and statistics that are essential for financial risk management. Every chapter is presented in a pedagogical manner, with associated Excel spreadsheets explaining the numerous practical examples.

CHAPTER 1 reviews the fundamental mathematical concepts: the symbols used and the basic rules of arithmetic, equations and inequalities, functions and graphs.

Foundations

Learning Outcome Statement
The candidate should be able to:

- Describe Rules of algebraic operations
- List the Order of algebraic operations
- Characterize Sequences
- Characterize Series
- Characterize Exponents
- Characterize Logarithms
- Characterize Exponential function and Natural Logarithms
- Solve Linear equalities and inequalities in one unknown
- Demonstrate the Elimination method
- Demonstrate the Substitution method
- Characterize Functions and Graphs
- Demonstrate continuous compounding
- Differentiate between discrete compounding and continuous compounding

CHAPTER 2 introduces the descriptive statistics that are commonly used to summarise the historical characteristics of financial data: the sample moments of returns distributions, 'downside' risk statistics, and measures of covariation (e.g. correlation) between two random variables.

Descriptive Statistics

Learning Outcome Statement

The candidate should be able to:

- Describe various forms of Data
- Discuss Graphical representation of data
- Explain the concept of The Moments of a Distribution
- Define, Discuss and Calculate the Measures of Location or Central Tendency
- Define, Discuss and Calculate the Measures of Dispersion
- Calculate Historical Volatility from Returns Data
- Define, Discuss and Calculate Skewness
- Define, Discuss and Calculate Kurtosis
- Describe Bivariate Data
- Discuss Covariance and Covariance Matrix
- Discuss Correlation Coefficient and Correlation Matrix
- Calculate the volatility of a portfolio

CHAPTER 3 focuses on differentiation and integration, Taylor expansion and optimisation. Financial applications include calculating the convexity of a bond portfolio and the estimation of the delta and gamma of an options portfolio.

Calculus

Learning Outcome Statement

The candidate should be able to:

- Differentiate between Differential Calculus and Integral Calculus
- Explain the concept of differentiation
- Demonstrate the application of the rules of differentiation to polynomial, exponential and logarithmic functions
- Calculate the modified duration of a bond
- Discuss Taylor Approximations
- Demonstrate the concept of convexity
- Demonstrate the concept of delta and gamma
- Demonstrate Partial Differentiation
- Demonstrate Total Differentiation
- Discuss the Fundamental Theorem of Analysis
- List the Indefinite Integral(s) of function(s)
- List the Rules of Integration
- Discuss Optimisation of Univariate and Multivariate functions
- Demonstrate Constrained Optimisation using Lagrange Multipliers

CHAPTER 4 covers matrix operations, special types of matrices and the laws of matrix algebra, the Cholesky decomposition of a matrix, and eigenvalues and eigenvectors. Examples of financial applications include: manipulating covariance matrices; calculating the variance of the returns to a portfolio of assets; hedging a vanilla option position; and simulating correlated sets of returns.

Linear Mathematics and Matrix Algebra

Learning Outcome Statement

The candidate should be able to:

- Demonstrate basic operations of Matrix Algebra
- Solve two Linear Simultaneous Equations using Matrix Algebra
- Demonstrate Portfolio Construction
- Demonstrate Hedging of a Vanilla Option Position
- Describe Quadratic Forms
- Discuss the Variance of Portfolio Returns as a Quadratic Form
- Define Positive Definiteness
- Demonstrate Cholesky Decomposition
- Demonstrate Eigenvalues and Eigenvectors
- Describe Principal Components

CHAPTER 5 first introduces the concept of probability and the rules that govern it. Then some common probability distributions for discrete and continuous random variables are described, along with their expectation and variance and various concepts relating to joint distributions, such as covariance and correlation, and the expected value and variance of a linear combination of random variables.

Probability Theory

Learning Outcome Statement

The candidate should be able to:

- Explain the concept of probability
- Describe the different approaches to defining and measuring probability
- Demonstrate the rules of probability
- Define the discrete and continuous random variable
- Describe the probability distributions of a random variable
- Describe Probability density functions and histograms
- Describe the Algebra of Random variables
- Define the Expected Value and Variance of a discrete random variable
- Describe the Algebra of Continuous Random Variables
- Demonstrate Joint Probability Distributions
- Discuss covariance and correlation
- Discuss the Binomial Distribution
- Demonstrate the Poisson Distribution
- Describe the Uniform Continuous Distribution
- Discuss the Normal Distribution
- Discuss the Lognormal Probability Distribution and its use in derivative pricing
- Discuss the Student's t Distribution
- Discuss the Bivariate Normal Joint Distribution

CHAPTER 6 covers the simple and multiple regression models, with applications to the capital asset pricing model and arbitrage pricing theory. The statistical inference section deals with both prediction and hypothesis testing, for instance, of the efficient market hypothesis.

Regression Analysis

Learning Outcome Statement

The candidate should be able to:

- Define Regression Analysis and the different types of regression
- Demonstrate Simple Linear Regression
- Demonstrate Multiple Linear Regression
- Discuss the evaluation of the Regression Model
- Describe Confidence Intervals
- Describe Hypothesis Testing
- Demonstrate Significance Tests for the Regression Parameters
- Demonstrate Significance Test for R2
- Describe Type I and Type II Errors
- Demonstrate the concept of Prediction
- Describe the OLS Assumptions and main breakdowns of them
- Describe Random Walks and Mean Reversion
- Describe Maximum Likelihood Estimation

CHAPTER 7 looks at solving implicit equations (e.g. the Black-Scholes formula for implied volatility), lattice methods, finite differences and simulation. Financial applications include option valuation and estimating the 'Greeks' for complex options.

Numerical Methods

Learning Outcome Statement

The candidate should be able to:

- Demonstrate the Bisection method for solving Non-differential Equations
- Demonstrate the Newton-Raphson method for solving Non-differential Equations
- Describe the application of Goal Seek equation solver in Excel
- Demonstrate Unconstrained Numerical Optimisation
- Demonstrate Constrained Numerical Optimisation
- Demonstrate Binomial Lattices for valuing options
- Demonstrate Finite Difference Methods for valuing options
- Demonstrate Simulation using Excel

STUDY QUESTIONS

CALCULUS

Rate of Change

Q: Find the derivative of y=2x using rate of change approach:

$\Delta y = f(x+\Delta x) - f(x) = 2(x+\Delta x) - 2x$
$\Delta y = 2\Delta x$
$\Delta y/\Delta x = 2$
n the limit dy/dx = 2

Area/Volume

Q: Calculate the area under the curve $y - x^2$ for the range zero to one

$$Area - \int_0^1 y dx - \int_0^1 x^2 d^X = \left[\frac{x^3}{3}\right]_0^1 = ⅓$$

Optimization

Q: What is the maximum of the expression $5x + 2x^2 + 4y$ subject to the constraint that $2x + y = 20$?

In this example $f(x, y) = 5x + 2x^2 + 4y$ and $g(x, y) = 2x + y - 20$. (Note the reorganisation that is needed so that the constraint is in the form $g(x, y) = 0$.) The Lagrangian is then $L(x, y, \lambda) = 5x + 2x^2 + 4y - \lambda(2x + y - 20)$.

Differentiating:

$$\frac{\partial L}{\partial x} = 5 + 4x - 2\lambda\,; \qquad \frac{\partial L}{\partial x} = 4 - \lambda\,; \qquad \frac{\partial L}{\partial x} = 2x + y - 20.$$

Setting each to zero gives

$$5 + 4x - 2\lambda = 0 \qquad 4 - \lambda = 0 \qquad 2x + y - 20 = 0.$$

Solving gives $x = 0.75$, $y = 18.5$ and $\lambda = 4$. Thus the constrained maximum is $f(0.75, 18.5) = 78.875$.

Ordinary and Partial Derivatives

Q: Find a linear polynomial p(x) that is a tangent-line approximation for the function $f(x) = e^{2x - 4}$ at the point $x_0 = 3$.

a) $14.778x - 36.945$
b) $7.389x + 2.718$
c) $2.718x$
d) $14.778x + 0.018$

The tangent to this function will pass through the same value as the function at x=3 and will also have the same gradient. So we use the first derivative of the function to establish the slope of the tangent and use the normal $y=mx+c$ representation of a straight line. $2x-4$, at the point $x_0=3$, is worth 2, 2.718^2 is worth 7.39. The tangent at the point 3, is sloping at $2*e^{(2*3-4)}=14.778$: this discards b) and c). The d) line, at the point 3, is 44.35: too high, only a) remains. To double-check, $14.778x - 36.945 = 7.39$: a).

Integration

Q: Evaluate the definite integral: $\int_0^2 xe^{x^2}\,dx$

a) 5.437
b) 26.799
c) 21.285
d) 7.389

This integral can be done by noting that e^{x^2} differentiates to (using the chain rule) $2xe^{x^2}$ so the integral I = $\frac{e^{x^2}}{2}$ which on putting in the limits I = $\left[\frac{e^4}{2} - \frac{1}{2}\right]$ = 26.799, so answer b).

Matrix Algebra

Q: Determine the inverse matrix of: $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

a) $\begin{pmatrix} 1 & 1 \\ 0 & 0.5 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -1 \\ 0 & 0.5 \end{pmatrix}$ c) $\begin{pmatrix} 0.5 & -1 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & -1 \\ 0.5 & 0 \end{pmatrix}$

It is quicker to eliminate the matrices by multiplying them by the first matrix, as the product needs to be the identity matrix if it is the inverse. We can eliminate a), as it has no negative element. The matrix b) works. The other two matrices would fail to produce the desired Id matrix (as c) fails on first line first column, d) second line first column): b).

Positive Definiteness

Q: Under what circumstances is the 2 x 2 real symmetric matrix, A, positive definite?

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

In order for A to be positive definite, a and b both have to be positive and the determinate of the matrix must be positive.

Eigenvectors and Eigenvalues

Q: Show that the following vectors v1, and v2 are eigenvectors of A – what are the eigenvalues?

$$v1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad v2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad A = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$$

$$A * v1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} = v1, \quad eigenvalue = 1$$

$$A * v2 = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4 * v2, \quad eigenvalue = 4$$

$$v1 \neq k * v2$$

The eigenvectors are linearly independent.

Cholesky Factorization

Q: Perform the Cholesky factorisation on the following correlation matrix:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} = \begin{pmatrix} n_{11} & 0 \\ n_{12} & n_{22} \end{pmatrix} * \begin{pmatrix} n_{11} & n_{12} \\ 0 & n_{22} \end{pmatrix} = \begin{pmatrix} n_{11}^2 & n_{11}\, n_{12} \\ n_{11}\, n_{12} & n_{12}^2 + n_{22}^2 \end{pmatrix}$$

By equating elements of the matrix and eliminating terms, we get:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \rho & (1-\rho^2)^{½} \end{pmatrix} * \begin{pmatrix} 1 & \rho \\ 0 & (1-\rho^2)^{½} \end{pmatrix}$$

Random Variables

Q: What can we say about the sum $X + Y$ of two independent normal random variables X and Y:

a) It is normal only if X and Y have the same mean.
b) It is always normal
c) It is chi-squared
d) It is chi-squared if X and Y both have mean 0

One can refute c) and d) by taking a normal distribution with a zero standard deviation (it is just a number): add this to a normal distribution, and it will give a normal distribution. Adding two normal 0-variance distributions with different means, will give a normal distribution (with 0 variance, too), discarding a). A more elegant resolution is to remember the statistics course, to recall that is the sum of two normal independent distributions is itself a normal distribution, and go immediately to answer b).

Distributions and Densities

Q: What is the standard deviation of a random variable Q with probability function

$$Ø(q) = \begin{cases} .25 & q = 0 \\ .25 & q = 1 \\ .50 & q = 2 \end{cases}$$

a) .6875
b) .4727
c) .8291
d) .4281

We assume in the question that the distribution is discrete – so we calculate a simple mean equal to 0*0.25+1*0.25+0.5*2=1.25

Variance = $((1.25)^2)*0.25+((0.25)^2)*0.25+((0.75)^2)*0.5$

STD = SQRT (0.6875)=0.8291
So answer c).

Moments

Q: What is the formula for the skewness of a random variable X that has mean μ and standard deviation σ?

a) $\frac{E([x-\sigma]^2)}{\mu^2}$ b) $\frac{E([x-\mu]^4)}{\sigma^4}$ c) $\frac{E([x-\mu]^3)}{\sigma^3}$ d) $\frac{E([x-\mu]^4)}{E([x-\sigma]^4)}$

The solution d) can be eliminated as it makes little sense to use standard deviation in a sum for the divisor. It is worth remembering that skewness is the 3rd moment of a distribution. Above, a) looks like variance but is divided by the mean squared, b) is the 4th moment (kurtosis): so the skewness is answer c).

Covariance and Correlation Matrices

Q: A covariance matrix for a random vector:

a) Is strictly positive definite, if it exist
b) Is non-singular, if it exist
c) Always exists
d) None of the above

This question is full of red herrings. The main thing is not to get bogged down into what is a random vector, and to find out that the only relevant fact is that a co-variance, as the name indicates, is between two objects. Hence d).

Principal Component Analysis

Q: Why is PCA useful for risk management?

PCA allows the hedging to be carried out with a reduced number of hedge instruments as it allows the "normal" models of the risk to be identified and hedged directly rather than using bucketing or position-by-position approach.

Monte Carlo Simulation

Q: How can a random number generating function be used to generate samples from a normal distribution?

By using a sum of a large number of independent random numbers from a uniform distribution such as is generated by a random number function, it is possible to approximate a normal distribution. Usually twelve samples is considered large enough, so our normal random variable is the sum of twelve random numbers minus the mean (6).

Linear Regression

Q: If the regression coefficient b in the equation y = a +bX is less than one, with sample size = n, then the R^2 is:

a) Equal to sqrt(b)
b) Equal to the regression sum of squares / total sum of squares
c) Total sum of squares / n
d) (Total sum of squares * n) / (regression sum of square * (n-1))

$$R^2 = \frac{\text{the regression sum of squares}}{\text{the total sum of squares}} = \frac{\text{ESS}}{\text{TSS}}, \text{ (d).}$$

Basic Statistical Tests

Q: Which of the following would not be a typical statement subject to hypothesis testing?

a) Trader A generates positive alpha
b) Bond C trades at a positive spread to Bond Q
c) Stock X is a good investment
d) Volatility of Currency N is lower than Currency F

c) is a subjective assessment and is thus not suitable for hypothesis testing. However, c) could be rephrased to say that Stock X has a higher return and lower volatility than Stock Z, which is an objective statement.

PRM Self-Study Resources

The Professional Risk Managers' International Association

To meet the demand from PRM candidates for distance learning opportunities, PRMIA has developed a suite of self-study options to make studying for the PRM exam convenient and effective.

ICMA CENTRE INTERACTIVE STUDY GUIDE

PRMIA, in partnership with The ICMA Centre, University of Reading, offers a complete personal learning training package for the PRM exams I, II, III and IV featuring leading faculty members like Carol Alexander, Jacques Pezier, Salih Neftci, Moorad Choudhry, John Board, and others. This access will not expire and is for your personal use on PC or laptop, and the video can even be viewed on your iPod. This package includes:

- Studio-recorded lectures on DVD
- Adobe Captivate demonstrations of Excel workbooks
- A set of fully worked examples to accompany the lectures

EPRM COACH ONLINE TRAINING

ePRM Coach is a comprehensive self-study guide for the PRM™ Certification Exam. Designed in accordance with the PRM™ exam structure, the ePRM Coach is equipped with concepts and practices, including:

- Exhaustive theoretical material supplemented with contemporary case studies
- Learner friendly courses complete with formulae, definitions, concise summaries, and interactive simulations
- State-of-the-art simulated learning environment
- Solved examples, practice exercises and quizzes
- Mock exams from a proprietary database
- Timed tests in exam format
- Personalized results for self assessment
- Glossary, FAQs, tips center and pocket reference
- Valuable reference extracts
- Online access with 24x7 customer support

EPRM DIAGNOSTIC EXAMS

ePRM Diagnostic Exams are mock exams designed in accordance with PRMIA's exam structure and feature:

- Simulation of the actual PRM™ Exam
- State-of-the-art learning environment
- Timed tests in exam format
- Personalized results for self-assessment
- 24x7 online access

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