

Certificate in Quantitative Finance Learning Pathway

Lectures, preparatory and further reading, additional recorded material

Notes:

- 1. Lecture order and content may occasionally change due to circumstances beyond our control. However this will never affect the quality of the programme*
- 2. The "Follow-up recordings" are recommendations, please watch out for new Lifelong Learning lectures since these are being added to regularly*

Module 1: Basic Building Blocks of Finance Theory and Practice

It will be necessary to bring all students up to the same technical level. Most students will be familiar with the contents of this first module, but any gaps in a student's background will be identified and appropriate private study recommended. We introduce the rules of applied Itô calculus as a modelling framework. Simple stochastic differential equations and their associated Fokker-Planck and Kolmogorov equations are introduced. The random nature of asset price movements is considered. Discrete time random walks are introduced and the continuous-time lognormal random walk is obtained by rescaling and passing to a limit.

- Mathematics preliminaries: Review of ordinary calculus, Taylor's theorem in one and several variables, ordinary differential equations and the diffusion equation.
- Fokker-Planck and Kolmogorov equations, similarity solutions.
- Probability preliminaries: Review of discrete and continuous random variables, transition density functions, moments and important distributions, the Central Limit Theorem.
- Applied Itô calculus: Discrete time random walks, continuous Wiener processes via rescaling and passing to the limit. Quadratic variation, elementary Itô integrals and Itô's lemma.
- The random nature of prices: Examination of data, unpredictability, the need for probabilistic models, drift and volatility.
- Simulations: The lognormal random walk, probability density functions.
- Visual Basic workshop: The tools for implementing the models.

Preparatory reading:

- P. Wilmott, *Paul Wilmott Introduces Quantitative Finance*, second edition, 2007, John Wiley. Chapters 4, 5, 7
- M. Jackson and M. Staunton, *Advanced Modelling in Finance Using Excel and VBA*, 2001, John Wiley. Chapters 1 - 4

Further reading:

- G.R. Grimmett and D.R. Stirzaker, *Probability and Random Processes*, 1997, Oxford University Press
- J.D. Hamilton, *Time Series Analysis*, 1994, Princeton University Press
- J.A. Rice, *Mathematical Statistics and Data Analysis*, 1988, Wadsworth-BrooksCole
- S.N., *An Introduction to the Mathematics of Financial Derivatives*, 1996, Academic Press

Follow-up recording(s), extra lecture(s):

- Linear Algebra
- Stochastic Calculus
- Differential Equations
- Fundamentals of Optimization

Lecture 1.1:

- Notation commonly used in mathematical finance
- How to examine time-series data to model returns
- The random nature of prices
- Unpredictability
- The need for probabilistic models
- The Wiener process, a mathematical model of randomness
- A simple model for equities, currencies, commodities and indices

Lecture 1.2:

- Taylor series
- A trinomial random walk
- Transition density functions
- Our first differential equation
- Similarity solutions

Lecture 1.3:

- The Central Limit Theorem
- The meaning of Markov and martingale
- Brownian motion
- Stochastic differential equations
- Itô's lemma

Lecture 1.4:

- Continuous-time stochastic differential equations as discrete-time processes
- Simple ways of generating random numbers in Excel
- Correlated random walks
- Using Itô's lemma to manipulate stochastic differential equations

Lecture 1.5:

- Visual Basic workshop
- Hands-on implementation, an introduction

Lecture 1.6: Methods for Quantitative Finance: I

- Double Integration & Applications
- Probability Distributions

Module 2: Risk and Return

This unit deals with the classical portfolio theory of Markowitz, the Capital Asset Pricing Model, more recent developments of these theories, also option types and strategies.

- Simulations: The lognormal random walk, probability density functions.
- Risk and reward: Measuring return, expectation and standard deviation.
- Modern Portfolio Theory (Markowitz): Expected returns, variances and covariances, benefits of diversification, the opportunity set and the efficient frontier, the Sharpe ratio, and utility functions.
- Capital Asset Pricing Model: Single-index model, beta, diversification, optimal portfolios, the multi-index model.
- Value at risk: Profit and loss for simple portfolios, tails of distributions, Monte Carlo simulations and historical simulations, stress testing and worst-case scenarios. Portfolios of derivatives.
- Financial markets and products: Bonds, equities, currencies, commodities and indices.
- Introducing futures, forwards and options: Simple contingent claims, definitions and uses.
- Review of option strategies: Building up special payoff structures using vanilla calls and puts, horizontal, vertical and diagonal spreads.
- Review of options as speculative investments: Taking a view, gearing, strategies that benefit from moves in the asset or in volatility.
- The binomial model: Up and down moves, delta hedging and self-financing replication, no arbitrage, a pricing model, risk-neutral probabilities.
- Martingale theory: Fundamental definitions, concepts, results and tools.

Preparatory reading:

- P. Wilmott, *Paul Wilmott Introduces Quantitative Finance*, second edition, 2007, John Wiley. Chapters 1, 2, 3, 20-22
- M. Jackson and M. Staunton, *Advanced Modelling in Finance Using Excel and VBA*, 2001, John Wiley. Chapters 6—8, 10
- E.G. Haug, *The Complete Guide to Option Pricing Formulas*, second edition, 2007, McGraw-Hill Professional. Chapter 7

Further reading:

- S.N. Neftci, *An Introduction to the Mathematics of Financial Derivatives*, 1996, Academic Press
- E.J. Elton & M.J. Gruber, *Modern Portfolio Theory and Investment Analysis*, 1995, John Wiley
- R.C. Merton, *Continuous Time Finance*, 1992, Blackwell
- N. Taleb, *Dynamic Hedging*, 1996, John Wiley

Follow-up recording(s), extra lecture(s):

- Investment Lessons from Blackjack and Gambling
- Lagrange Optimization
- Quants Toolbox
- Symmetric Downside Sharpe Ratio
- Beyond Black-Litterman: Views on Generic Markets

Lecture 2.1:

- Measuring risk and return
- Benefits of diversification
- Modern Portfolio Theory and the Capital Asset Pricing Model
- The efficient frontier
- Optimizing your portfolio
- How to analyze portfolio performance
- Alphas and betas

Lecture 2.2:

- The time value of money
- Equities, commodities, currencies and indices
- Fixed and floating interest rates
- Futures and forwards
- No-arbitrage
- The definitions of basic derivative instruments
- Option jargon
- No arbitrage again and put-call parity
- How to draw payoff diagrams
- Simple option strategies

Lecture 2.3:

- The meaning of Value at Risk (VaR)
- How VaR is calculated in practice
- Simulations and bootstrapping
- Simple volatility estimates
- The exponentially weighted moving average

Lecture 2.4:

- A simple model for an asset price random walk
- Delta hedging
- No arbitrage
- The basics of the binomial method for valuing options
- Risk neutrality

Lecture 2.5:

- Martingale definitions and concepts
- Important results and tools

Lecture 2.6: Methods for Quantitative Finance: II

- Numerical Methods
- Stochastic Calculus

Module 3: Equity, Currency and Commodity Derivatives

The Black-Scholes theory, built on the principles of delta-hedging and no arbitrage, has been very successful and fruitful as a theoretical model and in practice. The theory and results are explained using different kinds of mathematics to make the student familiar with techniques in current use.

- The Black-Scholes model: A stochastic differential equation for an asset price, the delta-hedged portfolio and self-financing replication, no arbitrage, the pricing partial differential equation and simple solutions.
- The greeks: delta, gamma, theta, vega and rho and their uses in hedging.
- Risk-neutrality: Fair value of an option as an expectation with respect to a risk-neutral density function.
- Early exercise: American options, elimination of arbitrage, modifying the binomial method, gradient conditions, formulation as a free boundary problem.
- Elementary numerical analysis: Monte Carlo simulation and the explicit finite-difference method.
- Value at Risk: Portfolios of derivatives.
- Martingales: The probabilistic mathematics used in derivatives theory

Preparatory reading:

- P. Wilmott, Paul Wilmott Introduces Quantitative Finance, second edition, 2007, John Wiley. Chapters 6, 8, 27-30
- M. Jackson and M. Staunton, Advanced Modelling in Finance Using Excel and VBA, 2001, John Wiley. Chapters 9, 11—12
- E.G. Haug, The Complete Guide to Option Pricing Formulas, second edition, 2007, McGraw-Hill Professional. Chapters 1, 2, 7, 8, 12
- P. Wilmott, Paul Wilmott On Quantitative Finance, second edition, 2006, John Wiley. Chapter 12, 49, 50

Further reading:

- N. Taleb, Dynamic Hedging, 1996, John Wiley
- J.C. Hull, Options, Futures and Other Derivatives (5th Edition), 2002, Prentice-Hall
- K.W. Morton and D.F. Mayers Numerical Solution of Partial Differential Equations: An Introduction, 1994, Cambridge University Press
- G.D. Smith, Numerical Solution of Partial Differential Equations, 1985, Oxford University Press
- S.N. Neftci, An Introduction to the Mathematics of Financial Derivatives, 1996, Academic Press

Follow-up recording(s), extra lecture(s):

- American Options
- Infinite Variance

Lecture 3.1:

- The assumptions that go into the Black-Scholes equation
- Foundations of options theory: delta hedging and no arbitrage
- The Black-Scholes partial differential equation
- Modifying the equation for commodity and currency options
- The Black-Scholes formulae for calls, puts and simple digitals
- The meaning and importance of the Greeks, delta, gamma, theta, vega and rho
- American options and early exercise
- Relationship between option values and expectations

Lecture 3.2:

- The Greeks in detail
- Delta, gamma, theta, vega and rho
- Higher-order Greeks
- How traders use the Greeks

Lecture 3.3:

- The justification for pricing by Monte Carlo simulation
- Grids and discretization of derivatives
- The explicit finite-difference method

Lecture 3.4:

- The many types of volatility
- What the market prices of options tells us about volatility
- The term structure of volatility
- Volatility skews and smiles
- Exploiting your volatility models
- Should you hedge using implied or actual volatility?

Lecture 3.5:

- Martingale theory and its relevance to pricing
- Its role in practice
- Examples

Module 4: Interest Rates and Products

This module starts with a review of fixed income products and the simple but useful concepts of yield, duration and convexity, showing how they can be used in practice. The limitations of this approach and the need for a more sophisticated theory are explained. Many of the ideas seen in the equity derivatives world are encountered again here but in a more complex form.

- Fixed-income products: Fixed and floating rates, bonds, swaps, caps and floors.
- Yield, duration and convexity: Definitions, use and limitations, bootstrapping to build up the yield curve from bonds and swaps.
- Stochastic interest rate models, one and two factors: Transferring ideas from the equity world, differences from the equity world, popular models, data analysis.
- Data analysis: Choosing the best model
- Calibration: Fitting the yield curve in simple models, use and abuse.
- Convertible bonds: Conversion, callability, putability, random factors.
- Heath, Jarrow and Morton model: Modelling the yield curve.

Preparatory reading:

- P. Wilmott, Paul Wilmott Introduces Quantitative Finance, 2001, John Wiley. Chapters 14—19
- M. Jackson and M. Staunton, Advanced Modelling in Finance Using Excel and VBA, 2001, John Wiley. Chapters 14—16
- E.G. Haug, The Complete Guide to Option Pricing Formulas, second edition, 2007, McGraw-Hill Professional. Chapters 11, 14
- P. Wilmott, Paul Wilmott On Quantitative Finance, second edition, 2006, John Wiley. Chapters 31-33, 36, 37

Further reading:

- N. Taleb, Dynamic Hedging, 1996, John Wiley
- J.C. Hull, Options, Futures and Other Derivatives (5th Edition), 2002, Prentice-Hall

Follow-up recording(s), extra lecture(s):

- CB Coding Workshop
- Term Sheets

Lecture 4.1:

- Names and properties of the basic and most important fixed-income products
- Features commonly found in fixed-income products
- Simple ways to analyze the market value of the instruments: yield, duration and convexity
- How to construct yield curves and forward rates
- Swaps
- The relationship between swaps and zero-coupon bonds

Lecture 4.2:

- Stochastic models for interest rates
- How to derive the pricing equation for many fixed-income products
- The structure of many popular one-factor interest rate models
- The theoretical framework for multi-factor interest rate modeling
- Popular two-factor models

Lecture 4.3:

- How to choose time-dependent parameters in one-factor models so that
- Today's yield curve is an output of the model
- The advantages and disadvantages of yield curve fitting
- How to analyze short-term interest rates to determine the best model
- for the volatility and the real drift
- How to analyze the slope of the yield curve to get information about the market price of risk

Lecture 4.4:

- The basic Convertible Bond (CB)
- Market conventions for the pricing and analysis of CBs
- Converts as options
- CB arbitrage
- Pricing convertibles

Lecture 4.5:

- Heath, Jarrow and Morton model
- Evolution of the entire yield curve
- Risk neutrality

Module 5: Credit Products and Risk

Credit risk plays an important role in current financial markets. We see the major products and examine the most important models. The modeling approaches include the structural and the reduced form, as well as copulas.

- Credit risk and credit derivatives: Products and uses, credit derivatives, qualitative description of instruments, applications
- CDS pricing, market approach: Implied default probability, recovery rate, building a spreadsheet on pricing approach, building a spreadsheet on default timing, illustration of a working CDS pricing model
- Synthetic CDO pricing: The default probability distribution, default correlation, tranche sensitivity, pricing spread
- Risk of default: The hazard rate, implied hazard rate, stochastic hazard rate, utility theory, credit rating and Markov processes, credit derivatives.
- Transition matrices: Modelling change of rating, dynamics.
- Copulas: Uses for basket instruments, examples.

Preparatory reading:

- P. Wilmott, Paul Wilmott On Quantitative Finance, second edition, 2006, John Wiley. Chapters 39—42
- M.Choudhry, Structural Credit Products: Credit Derivatives and Synthetic Securitisation, 2004, John Wiley. Chapters 2-7, 10, 13

Further reading:

- A.K. Dixit and R.S. Pindyck, Investment Under Uncertainty, 1994, Princeton University Press
- J.C. Hull, Options, Futures and Other Derivatives (5th Edition), 2002, Prentice-Hall

Follow-up recording(s), extra lecture(s):

- Intensity Models: Theory, Calibration, Pricing

Lecture 5.1:

- Definitions and uses of credit derivatives
- Examples of credit derivatives, including term sheets

Lecture 5.2:

- Modeling default risk
- Structural models
- Reduced-form models
- Credit ratings
- How to model change of rating

Lecture 5.3:

- Copulas
- Mathematical definitions
- Uses for credit risk on baskets
- Examples

Lecture 5.4:

- Implied default probability
- Recovery rate
- Building spreadsheets for pricing and default timing
- Illustration of a working CDS pricing model

Lecture 5.5:

- Synthetic CDO pricing
- The default probability distribution
- Default correlation
- Tranche sensitivity

Module 6: Advanced Topics

The lognormal random walk and the Black-Scholes model have been very successful in practice. Yet there is plenty of room for improvement. The benefits of new models will be discussed from theoretical, practical and commercial viewpoints. When pricing complex products it is necessary to be able to correctly value vanilla products. Modern models adopt frameworks that ensure that basic products are perfectly calibrated initially. The models derived in earlier parts of the course are only as good as the solution. Increasingly often the problems must be solved numerically. We explain the main numerical methods, and their practical implementation.

- Volatility surfaces: Analysis and calibration, the behaviour of implied volatility.
- Stochastic volatility: Modelling and empirical evidence, pricing and hedging, mean-variance analysis.
- Jump diffusion: Discontinuous price paths, the Merton model, jump distributions, expectations and worst-case analysis.
- Exotic options: Common OTC contracts and their mathematical analysis.
- Monte Carlo simulations: Use for option pricing, speculation and scenario analysis, differences between equity/currency/commodity and the fixed-income worlds, accuracy, variance reduction, bootstrapping.
- Quasi-Monte Carlo methods: Low-discrepancy series for numerical quadrature, Halton, Sobol, Faure and Haselgrove methods.
- Finite-difference methods: Crank-Nicolson, and Douglas multi-time level methods, convergence, accuracy and stability.
- Non-probabilistic models: Uncertainty in parameter values versus randomness in variables, non-Brownian processes, nonlinear diffusion equations.
- Static hedging: Hedging exotic target contracts with exchange-traded vanilla contracts, optimal static hedging.
- Brace, Gatarek and Musiela: The evolution of forward rates continued, the discrete-maturity case.

Preparatory reading:

- P. Wilmott, *Paul Wilmott On Quantitative Finance*, second edition, 2006, John Wiley. Chapters 22-29, 37, 45-48, 50, 51-53, 57, 73, 76-79, 82
- Jaeckel, *Monte Carlo Methods in Finance*, 2002, John Wiley. Chapters 1—14

Further reading:

- K.W. Morton and D.F. Mayers, *Numerical Solution of Partial Differential Equations: An Introduction*, 1994, Cambridge University Press
- G.D. Smith, *Numerical Solution of Partial Differential Equations*, 1985, Oxford University Press

Follow-up recording(s), extra lecture(s):

- How to Hedge
- Inverse Problems in Finance
- Advanced Brace, Gatarek and Musiela
- Transform Methods

Lecture 6.1:

- Characterizing exotic options
- Simulations and partial differential equations
- Examples

Lecture 6.2:

- The Poisson process for modelling jumps
- Hedging in the presence of jumps
- How to price derivatives when the path of the underlying can be discontinuous
- Modeling volatility as a stochastic variable
- How to price contracts when volatility is stochastic
- The market price of volatility risk

Lecture 6.3:

- The relationship between implied volatility and actual volatility in a deterministic world
- The difference between 'random' and 'uncertain'
- How to price contracts when volatility, interest rate and dividend are uncertain
- Non-linear pricing equations
- Optimal static hedging with traded options
- How non-linear equations make a mockery of calibration

Lecture 6.4:

- Advanced Monte Carlo techniques
- Variance reduction
- Low-discrepancy sequences

Lecture 6.5:

- Implicit finite-difference methods including Crank-Nicolson
- Douglas schemes
- Richardson extrapolation
- American-style exercise and exotic options
- About the explicit finite-difference method for two-factor models
- About the ADI and Hopscotch methods

Lecture 6.6:

- Brace, Gatarek, Musiela, the LIBOR market model
- Discrete interest rates

Lecture 6.7:

- The effect of hedging at discrete times
- Hedging error
- The real distribution of profit and loss
- How to allow for transaction costs in option prices

Certificate in Quantitative Finance Reading List

Books in bold are supplied. Greyed-out books are preliminary reading for those needing a refresher in basic mathematics and probability. Other books are recommended but not required.

Numbers as column headings refer to the module for which the books are relevant.
Numbers underneath ● refer to chapters.

Substantial discounts are available on most of these books from the wilmott.com bookshop.

	ISBN	1	2	3	4	5	6
P. Wilmott, Paul Wilmott Introduces Quantitative Finance, second edition, 2007, John Wiley	0470319581	● 4, 5, 7	● 1, 2, 3, 20-22	● 6, 8, 27-30	● 14-19		
M. Jackson and M. Staunton, Advanced Modelling in Finance Using Excel and VBA, 2001, John Wiley	0471499226	● 1-4	● 6-8, 10	● 9, 11-12	● 14-16		
E.G. Haug, The Complete Guide to Option Pricing Formulas, second edition, 2007, McGraw-Hill Professional	0786312408		● 7	● 1, 2, 7, 8, 12	● 11, 14		
P. Wilmott, Paul Wilmott On Quantitative Finance, second edition, 2006, John Wiley	0470018704			● 12, 49, 50	● 31-33, 36, 37	● 39-42	● 22-29, 37, 45- 48, 50, 51-53, 57, 73, 76-79, 82
M.Choudhry, Structural Credit Products: Credit Derivatives and Synthetic Securitisation, 2004, John Wiley	0470821191					● 2-7, 10, 13	
P. Jaeckel, Monte Carlo Methods in Finance, 2002, John Wiley	0470821191						● 1-14
E. Mendelson and F. Ayres, <i>Schaum's Outline of Calculus</i> , 1999, McGraw-Hill	0070419736	●					
R. Bronson, <i>Schaum's Outline of Differential Equations</i> , 1994, McGraw-Hill (Chapters 1-13)	0070080194	●					
P. Duchateau and D.W. Zachmann, <i>Schaum's Outline of Partial Differential Equations</i> , 1986, McGraw-Hill (Chapters 1-9)	0070178976	●					

	ISBN	1	2	3	4	5	6
M.R. Spiegel, <i>Schaum's Outline of Probability and Statistics</i> , McGraw-Hill (Chapters 1-4)	0071350047	●					
D.G. Zill and M.R. Cullen <i>Differential Equations with Boundary Value Problems</i> , Brooks Cole	0534380026	●					
G.R. Grimmett and D.R. Stirzaker, <i>Probability and Random Processes</i> , 1997, Oxford University Press	0198572220	●					
J.D. Hamilton, <i>Time Series Analysis</i> , 1994, Princeton University Press	0691042896	●					
J.A. Rice, <i>Mathematical Statistics and Data Analysis</i> , 1988, Wadsworth-BrooksCole	0534082475	●					
S.N. Neftci, <i>An Introduction to the Mathematics of Financial Derivatives</i> , 1996, Academic Press	0125153929	●	●	●			
E.J. Elton & M.J. Gruber, <i>Modern Portfolio Theory and Investment Analysis</i> , 1995, John Wiley	0471007439		●				
R.C. Merton, <i>Continuous Time Finance</i> , 1992, Blackwell	0631185089		●				
N. Taleb, <i>Dynamic Hedging</i> , 1996, John Wiley	0471152803		●	●	●		
J.C. Hull, <i>Options, Futures and Other Derivatives</i> (5th Edition), 2002, Prentice-Hall	0130465925			●	●	●	
K.W. Morton and D.F. Mayers <i>Numerical Solution of Partial Differential Equations: An Introduction</i> , 1994, Cambridge University Press	0521429226		●			●	
G.D. Smith, <i>Numerical Solution of Partial Differential Equations</i> , 1985, Oxford University Press	0198596502		●			●	
A.K. Dixit and R.S. Pindyck, <i>Investment Under Uncertainty</i> , 1994, Princeton University Press	0691034109				●		